De factoribus numerorum compositorum dignoscendis

Disquisitio Academica,

cujus partem quartam,

adnuente amplissima Fac. Philos.

Præside

CAROLO JOH. Ds HILL,

Math. Prof. Reg. & Ord.

pro summis in philosophia capessendis honoribus

exhibet

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Blekingus.

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LUNDÆ,

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Sit a) $p = 191$ et — unde ipsius p multipla et schemata electa

1) $\nu = 2$.

		multipla	schemata electa
$100)\underline{191}(1$	$(\overline{1})$		
$:91 = 1$	1	191	
$\overline{9})\quad (10$	2	200—9	$\binom{+II}{9.2}$
$-\overline{1}$	21	2101	

2) $\nu = 3$.

$191)\overline{1000}(5$	0		
$(180)\quad :45 = 4$	$\underline{1}$	1000—45	$\binom{-III}{11.4}$
$\overline{11})\quad (4$	4	4011	
$+\overline{1}$	17	17000—1,	$\binom{III}{1.17}$

3) $\nu = 4$.

$191)\overline{10000}(51$	0		
(136): res. $68 = 2$	$\underline{1}$		
$55)\quad (1$	2	20055,	$\binom{+IV}{13.3}$
$(52): \overline{13} = 4$	3	$3.10^4 - 13$,	
$\overline{3}$	14	140003,	$\binom{-IV}{3.14}$

4) $\nu = 5$.

			schemata (o)
$191)100000(520$ seu $\overline{680}(+3)$	0		
$(107): \overline{107} = 1$	$\underline{1}$		
$\overline{84})\quad (1$	1		
$(69): \overline{23} = 3$	2		
$\overline{15})\quad (1$	7	7,00015	$\binom{-V}{15.7}$
$(8): \overline{8} = 1$	9	*) $90000\overline{8}$	$\binom{+V}{8.9}$
$\overline{7})\quad (1$	16	$160000\overline{7}$	$\binom{-V}{7.16}$
$\overline{1}$	25	$250000\overline{1}$	$\binom{+V}{1.25}$
		unde et $100000\overline{4}$	$\binom{+VII}{4.1} = \binom{+VII}{4.1}$

*) Obs. $90000\overline{8}$ (seu 90000\8 sicuti Burman) loco $9.10^5 - 8$ seu 899992 brevitatis caussa scribimus. Obs. 2. Quia quotus primus (523) nullius est

b) $p = 197$ et 1) $\nu = 3$ 2) $\nu = 4$

$197 | \overline{1000} \;|\; 5 | 0$ $\qquad\qquad$ $197 | \overline{100000} \;|50 | 0$

$: 15 = 13 | 1$ $\qquad\qquad$ $150 = 1 | 1$

$\overline{2)} \quad (7 | \overline{1}3 \ldots 13002$ $\qquad\qquad$ $\overline{47)} \quad (3 | \overline{1}$ Sch.

$\overline{1} \qquad | 92$ Sch. $\binom{-III}{2.13}$ $\qquad\qquad$ $(45) : \quad \overline{9} = 5 | 4 \; \binom{+IV}{9.4}$

Quoniam vero $197 = 200 - 3$, schema illius erit $\binom{+II}{3.2}$, cujus quadratum $\binom{+IV}{9.4}$ censeri potest. Id quod fusius ex sequentibus theorematibus elucidatur, quorum ope ex duobus schematibus inventis tertium elicitur.

Theor. Si tum $\binom{+n}{\alpha.\beta}$, tum $\binom{+n_{,}}{\alpha_{,}.\beta_{,}}$, schemata divisoris cujusdem (p) fuerint, similia erunt

I) $\binom{n_{,,}}{\alpha_{,,}.\beta_{,,}}$, siquidem $n_{,,} = n_{,} - n$, $\alpha_{,,} \underset{p}{\backsim} \alpha_{,}\beta$ & $\beta_{,,} \underset{p}{\backsim} \alpha\beta_{,}$ fuerint;

II) $\binom{n_{,,,}}{\alpha_{,,,}.\beta_{,,,}}$, existente $n_{,,,} = n_{,} + n$, $\alpha_{,,,} \underset{p}{\backsim} \alpha\alpha_{,}$ & $\beta_{,,,} \underset{p}{\backsim} \beta\beta_{,}$.

Quod ut demonstremus, observandum est signum $\binom{+n}{\alpha.\beta}$, numero cuidam N appositum, idem valere, ac si numerus N sub forma $10^n . b + a$ poneretur, ipsiusque loco acciperetur $\alpha b + \beta a$. Est igitur universim $\binom{+n}{\alpha.\beta}.(10^n . b + a) = \alpha b + \beta a$, et ideo speciatim $\binom{+n}{\alpha.\beta}. (10^{n_{,}} . \beta_{,} - \alpha_{,}) = 10^{n_{,} - n} . \alpha\beta_{,} - \alpha_{,}\beta$, si nempe $b = 10^{n_{,} - n}\beta$ et $a = -\alpha_{,}$ accipiuntur. Quoniam vero $\binom{+n_{,}}{\alpha_{,}.\beta_{,}}$ ipsius p est schema, erit $10^{n_{,}}\beta_{,} - \alpha_{,}$, ideoque et $10^{n_{,} - n}\alpha\beta_{,} - \alpha_{,}\beta$ ipsius p multiplum, sicuti etiam $10^{n''}\beta_{,,} - \alpha_{,,}$, cum $\alpha_{,,} \underset{p}{\backsim} \alpha_{,}\beta$ et $\beta_{,,} \underset{p}{\backsim} \alpha\beta_{,}$ sit, et

usus, divisio in 10ν omitti potest, si hujus dividendi loco adhibetur residuum primum ex operatione antecendenti in 10 (vel 10μ) ductum.

$n_{\prime\prime} = n_{\prime} - n$; quoniam igitur $10^{n_{\prime\prime}}\beta_{\prime\prime} - \alpha_{\prime\prime}$ multiplum ipsius p sit, etiam hujus schema erit $\binom{+n_{\prime\prime}}{\alpha_{\prime\prime}.\beta_{\prime\prime}}$. Q. e. I:mum.

Deinde si accipiamus $b = -\alpha_{\prime}$, et $a = 10^{n_{\prime}+n}\beta_{\prime}$, erit $\binom{+n}{\alpha.\beta}\cdot(-\alpha_{\prime}\cdot 10^{n} + 10^{n_{\prime}+n}\beta_{\prime}) = -\alpha\alpha_{\prime} + \beta\beta_{\prime}\cdot 10^{n_{\prime}+n}$, ideoque $\beta\beta_{\prime}\cdot 10^{n_{\prime}+n} \backsim \alpha\alpha_{\prime}$, seu $\beta_{\prime\prime\prime}\cdot 10^{n_{\prime\prime\prime}} - \alpha_{\prime\prime\prime}$, multiplum ipsius p, cum $10^{n}(10^{n_{\prime}}\beta_{\prime} - \alpha_{\prime})$ ex schemate $\binom{+n_{\prime}}{\alpha_{\prime}.\beta_{\prime}}$ hoc idem sit. Erit igitur et $\binom{+n_{\prime\prime\prime}}{\alpha_{\prime\prime\prime}.\beta_{\prime\prime\prime}}$ ipsius p schema. Q. e. II:dum.

Hinc vero concluditur, ex schematibus $\binom{+n}{\alpha.\;}$ et $\binom{+n_{\prime}}{\alpha\;.\;}$, cum idem ac $\binom{+n}{\alpha.1}$ et $\binom{+n_{\prime}}{\alpha_{\prime}.1}$ valeant, componi posse $\binom{+(n_{\prime}+n)}{\alpha\alpha_{\prime}\;.\;}$ et tertii generis $\binom{+(n_{\prime}-n)}{\alpha_{\prime}\;.\;\alpha}$; similiterque $\binom{n_{\prime}+n}{.\beta\beta_{\prime}}$ et $\binom{+(n_{\prime}-n)}{\beta.\beta_{\prime}}$ ex $\binom{n_{\prime}+}{.\beta_{\prime}}$ atque $\genfrac{[}{]}{0pt}{}{n+}{.\beta}$; itemque $\genfrac{[}{]}{0pt}{}{n+\nu}{\alpha.\beta}$ ex $\genfrac{[}{]}{0pt}{}{+n}{\alpha.\;}$ atque $\genfrac{[}{]}{0pt}{}{\nu+}{.\beta}$.

Vides igitur, schema tertii generis ex primis et secundis inveniri posse; ubi observari licet, exponentem $(n_{\prime}+n)$, si justo major foret, minimo periodi decimalis numero minui posse. Sic ex. gr. ad $p = 149$ invenimus schemata $\binom{+XXXII}{5\;.\;}$ et $\binom{+XXXVIII}{7\;.\;}$, unde componimus $\binom{+VI}{7.5}$ atque $\binom{+LXX}{35\;.\;}$; similiter $\binom{+XLIII}{147\;.\;}$ seu $\binom{-XLIII}{2\;.\;}$ ex $\binom{+XXXVIII}{7\;.\;}$ et $\binom{+V}{21.\;}$; itemque $\binom{-LXXV}{10\;.\;}$ seu $\binom{-LXXIV}{1\;.\;}$ ex $\binom{-XLIII}{2\;.\;}$ atque $\binom{+XXXII}{5\;.\;}$, nec non ex eisdem componitur $\binom{-XI}{2.5}$; itemque ex $\binom{-XLIII}{2\;.\;}$ atque $\binom{+XXXVIII}{7\;.\;}$ tum $\binom{-V}{2.7}$, tum $\binom{-LXXXI}{14\;.\;}$ seu $\binom{+VII}{14\;.\;}$, quia $\binom{-LXXIV}{1\;.\;}$ alterum est schema, quod cum illo componi potest. Hinc vero patet, calculatione schema-

tum tertii generis, præcipueque tædiosa ipsius valoris exponentis (ν), universim supersedere nos posse; cum ista primi et secundi generis hæc suppeditent.

Attamen si tabula factorum numerorum millione minorum ad manus est, forsan commodius $\nu < 6$ accipitur, schemataque tertii generis modo jam edocto computantur; quin imo horum alterutrius periodus integra, cum inverso ordine procedant, sufficit. Quoniam enim $\binom{-XLIII}{2\ .\ }$ et $\binom{-LXXIV}{1\ .\ }$ ipsi 149 competunt, etiam $\binom{XXXI+}{\ .\ 2}$ huic competit.

Præterea vides, quomodo schemata computata (Ex. gr. divisorum *CCC* minorum, quorum tabulam videas in Diario D^ni^ Crelle T. XII. p. 35) invicem comprobari possint. Sic ex. gr. si ipsi 149 competunt $\binom{-V}{2.7}$ et $\binom{+VI}{7.5}$, erit tum $5.10^6 - 7$, tum $\binom{-V}{2.7}.(5.10^6 - 7)$ per 149 divisibile; id quod constat, quoniam $5.10^6 - 7 = 50.10^5 - 7$, ideoque $\binom{-V}{2.7}.(5.10^6 - 7) = \binom{-V}{2.7}.(50.10^5 - 7) = -2.50 - 7.7 = -149$. Similiter comprobes $7.10^5 + 2 \backsim 0$ et $\binom{+VI}{7.5}.(7.10^5 + 2).10 \backsim 0$.

Tabulæ vero schematum computatæ ope quivis numerus datus (N), num de factoribus *CCC* minoribus componatur, facile ex regulis jam antea explicatis inquiritur. Sint ex. gr.

Factorum	Schemata (seu regulæ)
67	$\binom{II-}{.\ 2}, \binom{-III}{5\ .\ }, \binom{IV+}{.\ 4}, \binom{+V}{5.2}, \binom{+IX}{9\ .\ }$.
73	$\binom{I}{.\ 22}, \binom{+II}{8.3}, \binom{-II}{3.8}, \binom{-III}{22\ .\ }, \binom{-IV}{1\ .\ }$
149	$\left[\begin{matrix}I+\\ .\ 15\end{matrix}\right], \left[\begin{matrix}+II\\ 2.3\end{matrix}\right], \left[\begin{matrix}-III\\ 3.7\end{matrix}\right], \left[\begin{matrix}IV-\\ .\ 35\end{matrix}\right], \left[\begin{matrix}-V\\ 2.7\end{matrix}\right], \left[\begin{matrix}VI+\\ .\ 22\end{matrix}\right], \left[\begin{matrix}+VI\\ 7.5\end{matrix}\right],$ $\left[\begin{matrix}-VIII\\ 9\ .\ \end{matrix}\right], \left[\begin{matrix}IX-\\ .\ 101\end{matrix}\right], \left[\begin{matrix}-X\\ 6\ .\ \end{matrix}\right], \left[\begin{matrix}X-\\ .\ 25\end{matrix}\right] = \left[\begin{matrix}-XII\\ 4\ .\ \end{matrix}\right]$ &c.

et $N = 1598|817$; erit

1) per 67, et a) $\left[\begin{smallmatrix} IV+ \\ .\ 4 \end{smallmatrix}\right] N \backsim 1598 + 4 . 1817 = 7268$

$N' = 88|66$

Et deinde $N'.\left[\begin{smallmatrix} II- \\ .\ 2 \end{smallmatrix}\right] = 88 - 2 . 66 = -44$

b) vel mox

$$\begin{array}{r} 1598|8|17 \left[\begin{smallmatrix} II- \\ .\ 2 \end{smallmatrix}\right] \\ -34 \\ \hline 1597|84 \\ -168 \\ \hline 14|29 \\ -58 \\ \hline -44 \end{array}$$

Ergo non $N \urcorner 67$, seu non $N \underset{67}{\backsim} 0$.

2) per 73 et a) $\left[\begin{smallmatrix} -IV \\ 1\ . \end{smallmatrix}\right] N = -1598 + 1817 = 219 = N'$

b) $\left[\begin{smallmatrix} +II \\ 8 . 3 \end{smallmatrix}\right] N'$ seu $\left[\begin{smallmatrix} +II \\ 8 . 3 \end{smallmatrix}\right] 219 = 8.2 + 3.19 = 73$; Ergo $N \urcorner 73$.

3) per 149; et a) $\left[\begin{smallmatrix} IV- \\ .\ 35 \end{smallmatrix}\right] N = 1568 - 35 . 1817 = -6200\overline{3} = N'$;
et deinde

b) $\left[\begin{smallmatrix} +II \\ 2 . 3 \end{smallmatrix}\right] 6200\overline{3} = 2 . 620 + 3 . \overline{3} = 1231$, et iterum

$\left[\begin{smallmatrix} +II \\ 2 . 3 \end{smallmatrix}\right] 1231 = 2 . 12 + 3 . 31 = 117$.

c) seu immediate $\left[\begin{smallmatrix} -III \\ 3 . 7 \end{smallmatrix}\right] . N' = 3 . 62 - 7 . \overline{3} = 207$ (non $\urcorner$ 149), ergo N non $\urcorner$ 149.

Hoc igitur modo numeri cujusvis factores, (saltem si < 300 fuerint, quo casu tabula nostra regulas compendiosas suppeditat), perquirendi sunt & dijudicandi. Sin vero tibi occurrerit factor aliquis major sæpius scrutandus, regulas, quibus facile agnoscatur, modo jam edocto ipse tibi elicies.

Sit Ex. gr. datus factor $p = 307$, cujus regulas cognoscere velis. Forma igitur primum fractionem decimalem $0{,}00326... = 1:307$

vulgari modo, inveniesque $\left\{\begin{matrix} \text{resid.} & 1, & 10, & 100, & 79, & -131, & -82; & \alpha_6 & \text{\&c.} \\ \text{ordin.} & 0, & 1, & 2, & 3, & 4, & 5, & 6 & \end{matrix}\right\}$.
Jam vero quia $82 = 79 + 3$, est $-\alpha_5 = \alpha_3 + 3 . \alpha_0$, & universim $\alpha_{5+n} = -(\alpha_{3+n} + 3 . \alpha_n)$; & igitur speciatim $\alpha_6 = -\alpha_4 - 3\,\alpha_1 = + 131 - 30 = 101$. Quia vero $101 = 100 + 1$, erit similiter $\alpha_6 = \alpha_2 + \alpha_0$ & $\alpha_{6+n} = \alpha_{2+n} + \alpha_n$, cujus formulæ ope seriem residuorum facillime continuabis, nempe

$$\left\{\begin{matrix} \text{res.} & -82, & 101, & 89, & -31, & -3, & -30, & 7, & 70, & 86, & -61, & 4, & 40, & 93, & 9 & \text{\&c.} \\ \text{ord.} & 5, & 6, & 7, & 8, & 9, & 10, & 11, & 12, & 13, & 14, & 15, & 16, & 17, & 18 & \text{\&c.} \end{matrix}\right\}$$

Inde vero constant schemata simplicia $\begin{bmatrix} \mathit{VI} \\ 101\,. \end{bmatrix}$, $\begin{bmatrix} -\mathit{IX} \\ 3\,. \end{bmatrix}$, $\begin{bmatrix} \mathit{XI} \\ 7\,. \end{bmatrix}$, $\begin{bmatrix} \mathit{XV} \\ 4\,. \end{bmatrix}$, $\begin{bmatrix} \mathit{XVIII} \\ 9\,. \end{bmatrix}$, ex quibus facile compones $\begin{bmatrix} -\mathit{II} \\ 7.3 \end{bmatrix}$, $\begin{bmatrix} \mathit{III} \\ 9.4 \end{bmatrix}$, $\begin{bmatrix} \mathit{IV} \\ 4.7 \end{bmatrix}$, &c. Hinc vero regulas quæsitas eo modo, qui in prima notula explicatus est, elicies. Sin vero nostra schemata tibi non satisfecerint, nova dicto modo calculabis; ut si ad 283 schema $\begin{pmatrix} +\mathit{IV} \\ 2\,.\,3 \end{pmatrix}$ minus arriserit, facile invenies hæc, quæ primum operandi modum exprimunt, $\begin{bmatrix} +\mathit{V} \\ 101. \end{bmatrix}$, $\begin{bmatrix} -\mathit{VIII} \\ 31\,. \end{bmatrix}$, $\begin{bmatrix} +\mathit{X} \\ 13. \end{bmatrix}$ &c. & hæc secundi generis: $\begin{bmatrix} \mathit{III} \\ .15 \end{bmatrix}$, $\begin{bmatrix} \mathit{V}- \\ .14 \end{bmatrix}$, $\begin{bmatrix} \mathit{VIII} \\ .73 \end{bmatrix}$ &c. atque ad tertium modum pertinentia: $\begin{bmatrix} \mathit{V}- \\ 81\,.\,2 \end{bmatrix}$, $\begin{bmatrix} \mathit{VI} \\ 39.2 \end{bmatrix}$, $\begin{bmatrix} \mathit{VI}- \\ 5.7 \end{bmatrix}$ &c.

Similiter ad divisores	schemata appertinentia invenies:
271	$\begin{pmatrix} -\mathit{III} \\ 84\,. \end{pmatrix}, \begin{pmatrix} -\mathit{IV} \\ 27\,. \end{pmatrix}, \begin{pmatrix} \mathit{I}- \\ .\,27 \end{pmatrix}, \begin{pmatrix} \mathit{II}- \\ .84 \end{pmatrix}, \begin{pmatrix} \mathit{II}-- \\ 13\,.\,8 \end{pmatrix}, \begin{pmatrix} \mathit{III} \\ 8\,.\,13 \end{pmatrix}, \begin{pmatrix} \mathit{II} \\ 3.19 \end{pmatrix}$.
277	$\begin{pmatrix} \mathit{II}- \\ .36 \end{pmatrix}, \begin{pmatrix} \mathit{IV} \\ 28. \end{pmatrix}, \begin{pmatrix} \mathit{V} \\ 3. \end{pmatrix}, \begin{pmatrix} -\mathit{VI} \\ 7\cdot 9 \end{pmatrix}, \begin{pmatrix} \mathit{VII}- \\ .12 \end{pmatrix}, \begin{pmatrix} \mathit{LXIX} \\ 1\,. \end{pmatrix}$.
291	$\begin{pmatrix} \mathit{IV} \\ 13.12 \end{pmatrix}, \begin{pmatrix} -\mathit{V} \\ 7.8 \end{pmatrix}, \begin{pmatrix} \mathit{VI} \\ 9\cdot 7 \end{pmatrix}, \begin{pmatrix} \mathit{VI} \\ .32 \end{pmatrix}, \begin{pmatrix} -\mathit{VIII} \\ 32. \end{pmatrix}, \begin{pmatrix} \mathit{XXVIII} \\ 1\,. \end{pmatrix}$.
293	$\begin{pmatrix} -\mathit{VI} \\ 9\cdot \end{pmatrix}, \begin{pmatrix} \mathit{IV} \\ 11.8 \end{pmatrix}, \begin{pmatrix} -\mathit{III} \\ 13.12 \end{pmatrix}, \begin{pmatrix} \mathit{VIII} \\ .14 \end{pmatrix}, \begin{pmatrix} \mathit{VII} \\ 2.13 \end{pmatrix}, \begin{pmatrix} -\mathit{LXXIII} \\ 1\,. \end{pmatrix}$.

Hoc modo sæpissime eveniet, ut regulæ computationem facilem præscribentes reperiantur. Interdum tamen accidere potest, ut vel exponens ν justo major faciendus sit, vel etiam ut numeri auxiliares α et β, etsi minores dimidio divisore semper sint, tamen satis magnos se præbeant. Sic ex. gr. ad divisorem 271 alteruter horum numerorum duobus constat zyphris (ut 13 vel 27 &c), nec minores, nedum si ν major accipiatur, invenire liceat. Quoniam hujus divisoris schema est $\binom{+V}{1\,.}$, et valores II, III, IV ipsius ν non nisi justo majores ipsorum α et β suppeditaverint valores, nihil omnino proderit majorem ipsius ν adhibere valorem. Si enim ex. gr. tentare velles $\nu = VII$, facile tibi persuaseris, eosdem inde orituros ipsorum α et β valores, ac qui ex positione $\nu = II$ inventi sunt, ex. gr. $\binom{VII}{3.19}$, quod schema ex illo $\binom{II}{3.19}$ juncto isti $\binom{V}{1\,.}$ immediate exoritur, quare hæc dua schemata unum idemque valent. Simile quid evenire potest ad plerosque factorum majorum p, qui valorem exponentis ν ex congruentia $10^{\nu} \underset{p}{\backsim} 1$ minimum suppeditaverint. Tali casu vel systema, secundum quod numeri designantur, mutandum est, vel alio modo hic factor p est dignoscendus. Hunc in finem sequentem modum proponere licebit: Calculentur multipla minora $2p$, $3p$, $4p$ et $5p$ ipsius p; quo facto horum id eligatur, quod numero dato N junctum summam vel differentiam præbeat, quæ multiplex decadis sit, et ultima zyphra, quæ ita oritur, deleta, novus numerus N' eodem modo tractetur. Sit ex. gr. $p = 271$ et $N = 11111$ numerus de factore p inquirendus. Ex regula proposita faciatur $N' = \frac{1}{10}(N - p) = 1111 - 27 = 1084$, et *)

$N'' = \frac{1}{10}(N' - 4p) = 108 - 108 = 0$; ideoque erunt N'', N' et N multipla ipsius p. Et universim facile evincitur, N multiplum ipsius p fore, quoties N' vel N'' &c. evanuerit; non vero multiplum, quoties alteruter horum numerorum minor ipso p inventus fuerit. Nam operatio, quam jam commendamus, revera aliqua est divisio reversa, ideoque si $N 7 p$ fuerit, tandem neccessario invenietur residuum $= o$; sin vero N' non $7 p$ fuerit, ad finem reperietur residuum $< p$ & $> o$: & vice versa si hujusmodi inventum fuerit residuum, N non multiplum ipsius p erit; sin vero residuum evanuerit, n multiplum ipsius p erit.

Ex. $N = 17689$ & $p = 271$ *)

$$
\begin{array}{r}
17689 \\
+271 \\ \hline
1796 \\
+1084 \\ \hline
288 \\
+542 \\ \hline
83
\end{array}
$$

$83 < p$; ergo N non $7 p$.

*)
$p = 271$
$2p = 542$
$3p = 813$
$4p = 1084$
$5p = 1355$

Ad finem hoc præterea nonnullis monendum forsan est, nos difficillimum illud problema de factoribus numeri dati (N) inveniendis tractare noluisse, quamquam aliquis ex titulo aliquid tale forsan exspectaverit. Nihilo tamen minus & nostra ad hoc problema solvendum aliquid faciat. Vulgari enim processu, ceteris exclusis, numeri reperiuntur primi, quorum alteruter ipsius N factor esse poterit; id quod per nostra schemata cito disquiritur. Quin imo ex observatione p. 16. & sq. proposita modus peti potest, cujus ope congruentia $a^x \underset{N}{\backsim} 1$ resolvatur, deindeque exponens ita inventus x ad factores ipsius N citius inveniendos adhiberi potest. Hæc vero res peculiarem sibi postulat tractationem, cui nullus hic jam datur locus.